Breeding objectives for pasture based dairy production systems

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Abstract

The aim of the study was to derive economic weights for milk production traits (milk, fat, and protein yield), survival, and mature body size for pastures based production systems. Economic weights were derived using a herd model and differentiating a profit function with respect to the traits of interest. Scaling was taken into account by assuming that the total feed supply per farm was constant and at an economically optimum level. Relative economic weights were expressed in genetic standard deviations. To investigate the robustness of the method to derive economic weights, economic weights were calculated for 11 different sets of herd parameters, varying production levels, average herd life, costs and returns. Protein yield had the highest relative economic weight, followed by survival and mature body size. The latter traits were approximately half as important as protein yield, with the economic weight for body size negative. Fat and milk yield were approximately equally important and 40% as important as protein yield. Milk (volume) yield had a negative economic weight. Economic weights were fairly robust to changes in herd parameters, and it was concluded that the method of calculating economic weights, using a herd model and constraining the total energy supply, was appropriate for pasture based production systems.

Keywords: Dairy cattle; Breeding objectives; Economic weights.

1. Introduction

In dairy cattle many traits are usually recorded on females, and estimated breeding values (EBV) calculated for the sires of those recorded cows. In general three types of recorded traits can be distinguished: production traits, type traits, and 'workability' traits. By workability traits we mean traits such as milking speed (MS) and temperament (TEMP), and generally traits other than production or type traits. In Australia, for example, 40 EBV are calculated for bulls locally progeny tested: 5 production EBV, 30 type EBV, and 5 workability EBV.

When selecting bulls and cows for breeding, an optimum combination of all EBVs is sought. A standard way of combining information on different traits for selection purposes is the selection index (Hazel, 1943). The theory and application in this study are restricted to the definition of the breeding objective, the determination of biological traits that are components of the breeding objective, and derivation of marginal economic values for those traits.

Several countries have implemented a selection index to combine EBV for milk (M), fat (F) and protein (Pr) yield (e.g. Dommerholt and Wilmink, 1986; Rozzi, 1991; Gibson et al., 1992). However, these indices are only based on the marginal returns and feed costs of the production traits, and do not take other important components of profit into account. For example, by ignoring the feed cost associated with maintenance requirement these selection indices favour...
high producing large cows over medium producing small cows.

There has long been a debate over how economic weights should be determined. The breeding objective is usually defined as profit from a particular farm business. Farm profit ($P$) depends on the genetic merit of the animals ($x$) and variables controlled by management ($m$), i.e.

$$P = P(x, m)$$

The conventional method of deriving economic weights ($a$) (e.g. Moav and Hill, 1966; Goddard, 1983) is to fix the management variables at the values which are optimum for the current genotype ($m_0$) and then examine the effect of a small change in $x$ about the current mean ($\bar{x}$), i.e.

$$a = \frac{\partial P}{\partial x}(\bar{x}, m_0),$$

where $\frac{\partial P}{\partial m}(\bar{x}, m_0) = 0$.

In practice most authors have chosen an arbitrary rather than optimum value for one management variable in order to fix the overall size of the farm business, e.g. number of cows. A major problem in deriving economic weights has been the choice of which variable to arbitrarily fix or constrain. For example, is it the number of milking cows which remains fixed, or total farm income?

McArthur (1987) and Amer and Fox (1992) argue that no constraint should be imposed but that the profit from genetic change should be calculated while simultaneously optimizing all variables controlled by management, including the scale of the enterprise. The existence of farms of widely different size suggests that the optimum for enterprise size is very flat. For example, Brascamp et al. (1985) and Smith et al. (1986) assume that returns and costs are proportional to enterprise size, implying that no optimum size exists and that economic weights are the same regardless of the scale of the enterprise chosen. In practice it may be more useful to have economic weights which are appropriate for the range of enterprise sizes actually occurring, rather than for a vaguely defined and unrealistic optimum size.

Smith et al. (1986) argue that benefits from an increase in the scale of production should not be counted as benefits from genetic change. Therefore economic weights should be calculated after discounting profit for changes in scale of enterprise. This is equivalent to calculating profit under the constraint that total costs or total returns remain constant. These relative economic weights were shown to be the same if the objective is returns/costs (R/C) (Smith et al., 1986). Also, if returns to management and capital are included as costs so that profit is zero, the same economic weights are obtained regardless of the constraint used (Brascamp et al., 1985). A practical difficulty with these methods is that all costs must be included, not just marginal costs, and some of the costs may be difficult to quantify, e.g. the cost of land.

Van Arendonk and Brascamp (1990) argue that economic weights should be calculated from product profitability, i.e. profit per unit of product, and show that this is equivalent to rescaling. However, as the authors acknowledge, specifying which linear combination of traits should be restricted in a multiple trait situation, i.e. which combination of traits is defined as a unit of product, may be difficult. For the example of fat and protein yield, Van Arendonk and Brascamp (1990) suggest that the weights given to these two traits in the product unit should 'reflect the ratio of products that is aimed for in the breeding programme'. This suggests a priori information about the economic weights and using that information to determine new economic weights.

Pasture is the principle source of feed for most Australian dairy farms. An obvious constraint, which is easily understood by farmers, is farm size i.e. hectares of land. Assuming that management variables such as fertilizer use are held constant then this implies that total feed available per farm is the constraint. In a pasture based feeding system a high proportion of total costs are related to farm size so holding total feed constant is almost equivalent to holding total costs constant as recommended by Smith et al. (1986). This approach has the additional advantage that it does not require estimation of some contentious costs such as the cost of pasture or the opportunity cost of capital invested. Hence, costs which are constant per farm (fixed costs) can be ignored.

Amer and Fox (1992) recommend that economic weights should be calculated while continuously re-optimizing the management variables. However, Goddard (1983) showed that, provided $\frac{\partial P}{\partial m}(\bar{x}, m_0) = 0$, the value of a small change in any trait is the same whether management variables are re-optimized or held at the optimum value for the current genotype.
Briefly re-stating the proof of Goddard (1983):

\[ P = P(x, m) \]

The optimum value of \( m \) depends on \( x \),

i.e. \( m = m(x) \). Profit after re-optimizing \( m \) is, therefore,

\[ P(x, m(x)) = P_0(x) \],

and the economic weight after re-optimizing is,

\[ \frac{\partial P_0}{\partial x} = \frac{\partial P}{\partial x} + (\frac{\partial P}{\partial m}) \frac{\partial m}{\partial x} \]

But, since \( \frac{\partial P}{\partial m(x, m_0)} = 0 \), it follows that,

\[ \frac{\partial P_0}{\partial m(x, m_0)} = \frac{\partial P}{\partial x}(\tilde{x}, m_0) \].

That is, the economic weight with re-optimization of \( m \) is the same as the economic weight with \( m \) held constant at the optimum value for the current genotype. This equivalence only applies when the effects of small changes in \( x \) (as calculated by partial differentials) are considered. It does not hold for larger changes in \( x \). For example, the value of a small change in involuntary culling is the same whether the level of voluntary culling is re-optimized or held at the present, optimum, value. However, this may not be true for larger changes in involuntary culling rate. Also, the equivalence does not hold if the optimum value \( m_0 \) occurs at a point where \( \frac{\partial P}{\partial m} (\tilde{x}, m_0) \neq 0 \). This can occur if the derivative \( \frac{\partial P}{\partial m} \) is not continuous, for example, a discontinuity may occur if there is a limit on the amount of milk which is paid for. Using the proof outlined above, we feel it is reasonable to calculate economic weights for a farm with fixed total feed and with other management variables held constant at current optimum values.

The aim of this study is to derive economic values for dairy traits when the breeding objective is defined as profit per farm with fixed total feed supply (demand) per farm. The restriction of total feed supply has not been applied before when deriving economic weights. We derive a general form of expressing economic weights when any restriction applies, and for discussing the economic weight of survival we derive general relationships between survival, stayability, and herd life.

2. Methods

2.1. Theory

Let profit \( (P) \) be income minus costs other than fixed costs. \( P \) is a function of traits \( x \) and total number of animals \( (N) \):

\[ P = P(N, x) \]

Similarly, assume the constraint \( (K) \) chosen is also a function of \( x \) and \( N \),

\[ K = K(N, x) \]

As a result of the constraint, \( N \) will be a function of \( x \), so

\[ P = P(N(x), x) = P_1(x) \]

[Note that the objective \( P_1 \) is not a function of \( N \), i.e. \( N \) is not a management variable in the sense used in the Introduction, because it is determined by \( x \) and the constraint that \( K \) is fixed. Nor is \( N \) assumed to be at an optimum value as management variables are. \( N \) is introduced merely because it simplifies the derivation of the economic weights.]

Economic weights are:

\[ \frac{\partial P_1}{\partial x} = \frac{\partial P}{\partial x} + (\frac{\partial P}{\partial N}) (\frac{\partial N}{\partial x}) \]  \hspace{1cm} (1)

Similarly,

\[ K = K(N(x), x) = K_1(x) \]

\[ \frac{\partial K_1}{\partial x} = \frac{\partial K}{\partial x} + (\frac{\partial K}{\partial N}) (\frac{\partial N}{\partial x}) = 0, \text{ so that} \]

\[ \frac{\partial N}{\partial x} = - (\frac{\partial K}{\partial x}) / (\frac{\partial K}{\partial N}) \]

Substituting this into Eq. (1) gives

\[ \frac{\partial P_1}{\partial x} = \frac{\partial P}{\partial x} - (\frac{\partial P}{\partial N})(\frac{\partial K}{\partial x}) / (\frac{\partial K}{\partial N}) \]  \hspace{1cm} (2)

In the special case where, \( P = N(R-C) \), where \( R \) is returns per animal, \( C \) is costs per animal, and the constraint is total cost (\( NC \)), then economic weights using (2) become,

\[ \frac{\partial P_1}{\partial x} = N[\frac{\partial R}{\partial x} - (\frac{\tilde{R}}{\tilde{C}})(\frac{\partial C}{\partial x})] \]

\[ = \frac{\partial P}{\partial x} - (\frac{\partial C}{\partial x})(\frac{\tilde{P}}{\tilde{C}}) \]

Brascamp et al. (1985) and Smith et al. (1986) show that these are the same economic weights as those derived by adjusting for changes in scale of enterprise cost, and the same relative economic weights as obtained by adopting economic efficiency, \( R/C \), as the objective.

Instead of constraining total costs to be constant, we constrain total feed used, i.e. Food \( (P) = \)
F(N,x) = constant, and exclude feed costs (including all costs which are fixed per farm) from the definition of P. Assuming that profit and food are proportional to the number of animals (because fixed costs are not included in P), Eq. (2) becomes,

\[ \frac{\partial P}{\partial x} = \frac{\partial F}{\partial x} + (\frac{\partial F}{\partial x})(\frac{\partial x}{\partial x}) (\frac{\partial P}{\partial x}) \]  

(3)

Eq. (3) means that the economic weight with the constraint is the economic weight ignoring the constraint minus the profit per unit feed requirement multiplied by the rate of change in feed requirement.

An alternative justification for Eq. (3) can be given. The definition of P includes all costs except feed costs which are handled by the constraint that they are fixed. An alternative objective would be \( P - \text{feed costs} \). If the cost of feed is \( \frac{P}{F} \), the economic weights are the same as those in Eq. (3). The assumption that the cost of feed is \( \frac{P}{F} \) means that mean profit after removing feed costs is zero as expected if all costs are included (Smith et al., 1986).

As shown by Smith et al. (1986) for the case of fixed total costs and taking R/C as the objective, an economic weight proportional to (3) is derived if the breeding objective is defined as \( (P/F) \). If in Eq. (3), \( \frac{\partial F}{\partial x} = 1 \) (i.e., we calculate an economic weight for trait F), then the economic weight is the same as that derived for traits under a quota constraint by Gibson (1989a). Our Eq. (3) is a more general form of Gibson’s derivation, extending the constraint (or quota) to specific input or output traits which are functions of traits receiving economic weights.

2.2. Dairy cattle application

Assumptions

Assume the following profit function for a dairy herd,

\[ P = N_m \sum_{k=1}^{3} (A_Ck \cdot a_k \cdot Y_k) \]

\[ + \sum_{i=0}^{k} (1 - S_i - D_i) N_i W_i p_i \]

\[ + N_{calf} (p_A A) P_{calf} \]

\[ - c_m N_m - (c_{g0} N_0 + c_{g1} N_1) \]

(4)

with: \( N_m \), \( N_i \) (= \( N_0 + N_1 \)), and \( \text{N_calf} \) are number of milking cows, number of replacement females, and number of sold calves respectively.

\( k \) is the total number of age classes, and \( N_i \) is the number of animals in age group \( i \). \( N_0 \) and \( N_1 \) are the number of animals in age class 0 (0–1 year olds) and 1 (1–2 year olds).

\( S_i \) and \( D_i \) are the relative survival \( (S_i = N_{i+1}/N_i) \) and proportion of deaths for age group \( i \).

\( Y_k \) \((k = 1, 3)\) is the mature equivalent production (production of 7-year old cows) in kg for milk \( (M) \), fat \( (F) \) and protein \( (Pr) \) respectively, and \( a_k \) \((k = 1, 3)\) is the payment price for 1 kg of \( M \), \( F \) and \( Pr \).

\( A_Ck \) \((k = 1, 3)\) is an average scaling factor for the milking herd, \( \frac{A_Ck = \sum (CF_k n_i)}{[\sum N_i]} \), for age group \( i = 2, k \). \( CF \) is a multiplicative correction factor relative to production of mature cows.

\( p_i \) and \( p_{calf} \) are the returns per kg liveweight from selling culled cows in age class \( i \) and surplus calves respectively. \( W_i \) is the average weight for age group \( i \). \( A \) is mature body size and \( p_A \) the calf sale weight as a proportion of \( A \). Hence \( (p_A \cdot A) \) is the average calf sale weight.

\( c_m, c_{g0}, c_{g1} \) are variable non-feed costs per milking cow and per replacement females in age classes 0 and 1 respectively.

\( O_i \) \((i = 1, 0)\) are other traits which are components of profit, such as MS, TEMP, type traits, and calving ease, and \( \text{c_calf} \) are the costs (returns) associated with those traits. Terms associated with other traits were included to present a general profit function.

The parameter \( \text{N_calf} \) may be written as \( \frac{N_m (1 - D_{-1}) - N_0}{N_m} \), where \( D_{-1} \) is the proportion of calves that die at birth or within a few weeks after birth.

We impose the restriction that the total energy requirement (or supply) is constant:

\[ F(N_i) + F(N_m) = \text{fixed} \]

For each age group, the total energy requirement (metabolisable energy (ME), in MJ per year) is divided into ME requirement for maintenance, growth, pregnancy and production:

\[ \text{ME} = \text{ME}_m + \text{ME}_g + \text{ME}_preg + \text{ME}_p \]
For each of the age groups 1 to k it is assumed that the energy requirement for pregnancy is a constant for all cows in those age groups, so that the total energy requirement for pregnancy is:

\[ ME_{\text{preg}} = \sum_{i=1}^{k} [N_i \times CP_i] \]

where CPi is the constant energy requirement per pregnancy for age group i.

\[ ME_{\text{p}} = N_m [AC_1 k_1 M + AC_2 k_2 F + AC_3 k_3 Pr] \]

with \( k_i \) constants (see, for example, Dommersolt and Wilming, 1986, and Beard, 1988, for values for \( k_i \)).

Final assumptions are that the weight of the animals at any time t can be described by a growth function \( g(A,t) \), where \( A \) is mature body weight, and that ME for maintenance and growth are functions of average weight (W) and live body weight gain (LWG) for a particular age group:

\[ ME_{m} = \sum N_i f_1(W_i), \quad ME_{g} = \sum N_i f_2(W_i, LWG_i) \]

The growth function we used, and its derivative with respect to mature body weight, is given in Appendix 1. Formulas and constants for functions \( f_1 \) and \( f_2 \) were obtained from SCA (1990). See Appendix 2 for a description of formulas and derivatives.

**Definition of survival traits**

We define survival in age class i, \( S_i = N_{i+1}/N_i \) as above, and stayability until age i, \( Stay_i = \prod_{j=i}^{k} S_j \).

Survival as a variable is only used for age groups corresponding to milking cows, because it is assumed that survival for calves (\( S_0 \)) and one-year olds (\( S_1 \)) is constant across the population, since the interest is in the longevity of cows in the milking herd. \( S_2 \) is the proportion of cows surviving from 1st to 2nd lactation. It follows that the average life in the milking herd (L) is the sum of all stayabilities, \( L = \sum Stay_i \) for \( i = 2, k \). Per definition \( Stay_1 = 1 \), \( S_k = 0 \), and the replacement rate (\( r \), \( r = 1/L \)).

If we treat all survivals as different traits, and assume \( N_m \) is constant, it follows that for \( i \geq 2 \),

\[ \partial L/\partial S_i = \left[1/S_i\right] \left[ \sum_{j=i}^{k} \text{Stay}_j \right] \]

\[ \partial r/\partial S_i = \left[-r^2/S_i\right] \left[ \sum_{j=i}^{k} \text{Stay}_j \right] \]

\[ \partial \text{Stay}_i/\partial S_i = (1/S_i) \text{Stay}_j \]

if \( i \leq j - 1 \)

= 0 otherwise

Then,

\[ \partial N_i/\partial S_i = N_m \left[ \text{Stay}_i \partial r/\partial S_i + r \partial \text{Stay}_i/\partial S_i \right] \]

\[ = \left[ - (N_m \text{Stay}_i r) / S_i \right] \left[ \sum_{j=i+1}^{k} \text{Stay}_j \right] - c_j \]

with \( c_j = 1 \) if \( i \leq j - 1 \)

= 0 otherwise

Also, \( \partial AC/\partial S_i = (1/N_m) \sum \partial N_i/\partial S_i \sum CP_j \),

\( \partial N_1/\partial S_i = \partial N_2/\partial S_i / S_1 \),

\( \partial N_0/\partial S_i = \partial N_2/\partial S_i / (S_0 S_1) \), and

\( \partial N_{\text{calf}} / \partial S_i = - \partial N_0 / \partial S_i \)

**Economic values**

Using previous results, economic weights can be derived for the traits which determine profit. These are given using first derivatives of profit with respect to traits of interest. However, a simple farm model has also been constructed based on the constraint that total feed is fixed. Economic weights have also been derived from this model by incrementing the mean of the trait of interest by a small amount and calculating the difference in profit, and results are the same as when differentiation is used.

1. Economic value for milk production traits. Using (3):

\[ \partial P_i/\partial Y_k = N_m AC_k \left[ a_k - k_k (\bar{F}_i/\bar{F}_1) \right] \]

(5)

Per cow, the economic value of a unit change in mature equivalent production of M, F or Pr, is the change in average production (less than unity because of age distribution) multiplied by the difference between the payment price and the cost of the additional feed energy required per unit of production, assuming the cost of feed is \( \bar{P}_i/\bar{F}_1 \).

2. Economic value for change in mature body size. The Eq. for P has terms related to mature body size (A) because it is assumed that returns from selling culled animals depends on their weight. Hence the economic value depends on an increase in returns from increased body weight, and an increase in energy requirement:
\[
\frac{\partial P}{\partial \Lambda} = \frac{\partial \bar{P}}{\partial \Lambda} - \frac{\partial \bar{F}}{\partial \Lambda} (\bar{F}/\bar{P}), \text{ with }
\frac{\partial \bar{F}}{\partial \Lambda} = \sum_{r=0}^{k} (1-S_i - D_j) N_{ip_i} \left( \frac{\partial W_j}{\partial \Lambda_i} \right) + N_{e\text{arf}} \frac{\partial P_{e\text{arf}}}{\partial \Lambda}
\]
\[
\text{and } \frac{\partial F_1}{\partial \Lambda} = \frac{\partial [ME_{m} + ME_{g}]}{\partial \Lambda}
\]

(6)

For both maintenance and growth, the derivatives are the weighted sums of the derivatives for each age class, for example,

\[
\frac{\partial ME_{m}}{\partial \Lambda} = \sum N_{f_i} f_i (W_i),
\]

with \( f' = \partial W/\partial \Lambda \) (see appendix 1)

(3) Economic value for survival. Survival of cows from one year to the next depends on the phenotype of the cow but also on management decisions taken by the farmer. Let

\[
S_i = S^*_i V_i,
\]

where \( (1-V_i) \) = proportion of cows in age group \( i \) voluntarily culled for low milk production, and \( (1-S^*_i) \) = proportion of cows in age group \( i \) involuntarily culled. The normal practice (e.g. Moav and Hill, 1966; Goddard, 1983) is to calculate economic weights for traits in the breeding objective (i.e. \( S^* \)) while holding management variables (i.e. \( V \)) constant at the optimum value for the current genotype. This means that changes in \( S^* \) do not affect the level of voluntary culling on milk production and so the gain in production between first and later lactations due to voluntary culling remains constant. This gain can be included in the age correction factors \( CF_i \). The age correction factors should account for the effect of voluntary culling on milk yield, which increases the yield from older cows, and genetic trend which decreases the yield from older cows relative to heifers. We have assumed that these two corrections cancel each other out. For example, if 5% of each age group are voluntarily culled on production each year, the gain is approximately 0.9% \( (r x r x \sigma = 0.11 \times 0.5 \times 16\% = 0.9\%), assuming a proportion selected of 0.95, a repeatability of 0.5 and a coefficient of variation of 16% \) per year, which is similar to the rate of genetic gain. The economic weight of involuntary culling \( S^*_i \) is,

\[
\frac{\partial P_i}{\partial S^*_i} = (\frac{\partial P_i}{\partial S_i}) \left( \frac{\partial S_i}{\partial S^*_i} \right) = \left( \frac{\partial P_i}{\partial S_i} \right) V_i
\]

And,

\[
\frac{\partial P}{\partial S_i} = N_m \left[ \sum_{k=1}^{3} \left( \frac{\partial \bar{A}_{ck}}{\partial S_i} \right) w_k Y_k \right] + \sum_{j=0}^{k} \left[ \left( 1-D_j \right) \left( \frac{\partial N_j}{\partial S_i} \right) - \frac{\partial N_j}{\partial S_i} \right] W_j p_j \right]
\]

\[
+ N_{e\text{arf}} \left( \frac{\partial P_{e\text{arf}}}{\partial S_i} \right) p_{e\text{arf}} - \left( \frac{\partial N_{e\text{arf}}}{\partial S_i} \right) c_r \]

(8)

\[
\frac{\partial F_i}{\partial S_i} = \sum_{j=1}^{k} \left( \frac{\partial N_j}{\partial S_i} \right) CP_j + N_m \sum_{j=1}^{3} \left( \frac{\partial \bar{A}_{ck}}{\partial S_i} \right) k_j Y_j + \sum \left( \frac{\partial N_j}{\partial S_i} \right) f_j (W_j) \]

\[
+ \sum \left( \frac{\partial N_j}{\partial S_i} \right) f_e (W_j, G_j)
\]

(9)

Economic weights for each survival trait can be derived using Eq. (3). If genetic change affects all survival scores equally, then the economic weight for survival is the sum of the economic weights of survival scores \( S_i \) to \( S_{i-1} \).

4) Economic value for other traits. From (4), and assuming that \( \frac{\partial F}{\partial O_i} = 0 \), it follows that the economic value for trait \( O_i \) is,

\[
\frac{\partial P}{\partial O_i} = -c_{o_i} N_m
\]

2.3. Numerical examples

Economic values for the production traits, mature body weight and survival were calculated using Eqs. (5) to (9). Economic weights for other traits will be considered in another study. Table 1 shows a set of constants used in all calculations, and in Table 3 different sets of parameters are presented. For each parameter set, economic values were calculated. Age correction factors (\( CF_i \)) were taken from Beard (1992), and average survival scores from Madgwick and Goddard (1985).
Table 1
Constant names (in bold) and constant values used for calculations of economic values

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<th>Energy Content</th>
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<td>Lactose proportion (%)</td>
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<th>S₆</th>
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<td>6-7</td>
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<td>8-9</td>
<td>9-10</td>
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<td>Survival (%)</td>
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<tr>
<th>Maintenance and Growth Coefficients</th>
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</thead>
<tbody>
<tr>
<td>k₁</td>
<td>0.0025</td>
</tr>
<tr>
<td>k₂</td>
<td>10.38</td>
</tr>
<tr>
<td>k₃</td>
<td>0.1138</td>
</tr>
<tr>
<td>k₄</td>
<td>2.14</td>
</tr>
<tr>
<td>k₅</td>
<td>6.7</td>
</tr>
<tr>
<td>k₆</td>
<td>20.3</td>
</tr>
<tr>
<td>Pₐ</td>
<td>0.0057</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Age correction factors for the default age distribution</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Volume, AC₁</td>
<td>0.901</td>
</tr>
<tr>
<td>Fat, AC₂</td>
<td>0.905</td>
</tr>
<tr>
<td>Protein, AC₃</td>
<td>0.901</td>
</tr>
</tbody>
</table>

The default parameter set (Table 2) corresponds to a farm with Holstein-Friesian (HF) cattle using present day average production and prices which were obtained from dairy factories and extention officers. Recall that per animal costs do not include feed costs. For pasture based production systems the per animal costs are, for example, costs for A.I., milk recording and veterinary treatment. Housing costs are very small for pasture based dairy enterprises. Scenarios 1 to 3 are used to investigate the effect of changes in the relative importance of milk, fat, and protein prices. Scenarios 4 and 5 reflect changes in cull stock prices and production costs, respectively, and scenarios 6 and 7 reflect changes in average milk production and mature size of the cows. In parameter set 8 both production and mature size are decreased relative to the default set. The comparison of scenario 8 to the default scenario could reflect farms stocked with different breeds, e.g. Jersey vs. Holstein. Scenarios 9 to 11 represent farm parameters more typical of Europe and North America, i.e. a lower average herd life (scenario 9), higher costs per cow (scenario 10) and much higher milk production (scenario 11). Differences in economic values from the different parameter sets reflect the sensitivity of the relative economic values to changes in parameters.

3. Results

Results for the set of default parameters and 11 alternative scenarios are presented in Table 4, with the economic weights in Australian dollars per cow per year (1 $ is approximately 0.7 US$). Standardised economic weights are also shown and are expressed in genetic standard deviation units (as in, for example, Goddard, 1985) relative to the standardised value for protein yield, so that they can be compared to each other. Assumptions about genetic standard deviations are shown in Table 3. In all calculations the economic value for survival is the sum of the economic values
Table 2
Parameter sets

<table>
<thead>
<tr>
<th></th>
<th>Default</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
<th>7</th>
<th>8</th>
<th>9</th>
<th>10</th>
<th>11</th>
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<tbody>
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<td><strong>Milk prices</strong></td>
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<td></td>
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</tr>
<tr>
<td>Fat ($ kg(^{-1}))</td>
<td>2.20</td>
<td>1.80</td>
<td></td>
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<td></td>
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<td>Protein ($ kg(^{-1}))</td>
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<tr>
<td>Volume ($ L(^{-1}))</td>
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<td>-0.10</td>
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<tr>
<td>Calves ($ kg(^{-1}))</td>
<td>1.00</td>
<td></td>
<td></td>
<td>1.20</td>
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<tr>
<td>Heifers ($ kg(^{-1}))</td>
<td>1.09</td>
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<td></td>
<td>1.31</td>
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<tr>
<td>Cows ($ kg(^{-1}))</td>
<td>0.83</td>
<td></td>
<td></td>
<td>1.00</td>
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<tr>
<td><strong>Annual production costs</strong></td>
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<tr>
<td>Calves ($ head(^{-1}))</td>
<td>25</td>
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<td>100</td>
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<tr>
<td>Heifers ($ head(^{-1}))</td>
<td>45</td>
<td></td>
<td></td>
<td>54</td>
<td>40</td>
<td>180</td>
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<tr>
<td>Cows ($ head(^{-1}))</td>
<td>100</td>
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<td></td>
<td>120</td>
<td>90</td>
<td>400</td>
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<tr>
<td><strong>Annual Production of Mature Cows</strong></td>
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<tr>
<td>Milk fat (kg)</td>
<td>200</td>
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<td></td>
<td></td>
<td>240</td>
<td>185</td>
<td>280</td>
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<td>Milk protein (kg)</td>
<td>160</td>
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<td></td>
<td></td>
<td>192</td>
<td>135</td>
<td>224</td>
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<td>Milk volume (L)</td>
<td>5000</td>
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<td></td>
<td></td>
<td>6000</td>
<td>3600</td>
<td>7000</td>
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<tr>
<td>Standard reference weight (kg)</td>
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<tr>
<td><strong>Herd life</strong></td>
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<td></td>
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<td></td>
<td></td>
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<tr>
<td>Average herd life (year)</td>
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<tr>
<td>Replacement rate (r)</td>
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</tbody>
</table>

$, Australian dollars throughout.

Table 3
Coefficients of variation (CV, in %) and heritabilities ($h^2$, in %) for traits in the breeding objective

<table>
<thead>
<tr>
<th>Trait</th>
<th>CV</th>
<th>$h^2$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Milk yield</td>
<td>19</td>
<td>25</td>
</tr>
<tr>
<td>Fat yield</td>
<td>19</td>
<td>25</td>
</tr>
<tr>
<td>Protein yield</td>
<td>18</td>
<td>25</td>
</tr>
<tr>
<td>Mature body weight</td>
<td>12</td>
<td>40</td>
</tr>
<tr>
<td>Survival in lactation i</td>
<td>$[(1-S_i)/S_i]^1$</td>
<td>2.5</td>
</tr>
<tr>
<td>Mean survival</td>
<td>$[r/(1-r)]^1$</td>
<td>2.5</td>
</tr>
</tbody>
</table>

r, replacement rate.

for the individual survival traits. The corresponding trait for the sum of survival economic weights is called 'mean survival'. For each scenario four additional results are given: average farm profit per unit of feed, the income from milk as a proportion of total income, the implicit value of a replacement heifer, and the value of a replacement heifer minus the salvage value of cull cows (net replacement cost (Allaire and Gibson, 1992)). For calculation of the implicit value of replacement heifers, see Appendix 3.

For the default scenario, a breakdown of the total feed consumption into separate components (maintenance, growth, production and pregnancy) is shown in Table 5. Maintenance and milk production make up nearly 90% of the total energy consumed by the herd.

Relative economic weights for volume, fat and protein yield are fairly robust to changes in input parameters, with protein yield always having the largest weight, usually approximately 2.5 times that of fat yield and milk volume (see Table 4). The main effect of reducing the value of fat yield (scenario 1) or increasing the value of protein yield (scenario 2) is that protein yield becomes more important relative to milk and fat.
Table 4
Economic weights for production traits, survival, and mature size

<table>
<thead>
<tr>
<th></th>
<th>Default</th>
<th>Parameter set</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>1</td>
<td>2</td>
</tr>
<tr>
<td>Economic weights ($\text{cow}^{-1}\text{ year}^{-1}$)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Milk fat (kg)</td>
<td>1.095</td>
<td>0.805</td>
</tr>
<tr>
<td>Milk volume (l)</td>
<td>-0.042</td>
<td>-0.041</td>
</tr>
<tr>
<td>Mature size (kg)</td>
<td>-0.582</td>
<td>-0.520</td>
</tr>
</tbody>
</table>

Standardised economic weights

<p>| | | | | | | | | | | | |</p>
<table>
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<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Milk fat</td>
<td>0.41</td>
<td>0.30</td>
<td>0.19</td>
<td>0.51</td>
<td>0.40</td>
<td>0.42</td>
<td>0.38</td>
<td>0.43</td>
<td>0.42</td>
<td>0.43</td>
<td>0.51</td>
</tr>
<tr>
<td>Milk protein</td>
<td>1.00</td>
<td>1.00</td>
<td>1.00</td>
<td>1.00</td>
<td>1.00</td>
<td>1.00</td>
<td>1.00</td>
<td>1.00</td>
<td>1.00</td>
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<td>1.00</td>
</tr>
<tr>
<td>Milk volume</td>
<td>-0.40</td>
<td>-0.38</td>
<td>-0.30</td>
<td>-0.90</td>
<td>-0.40</td>
<td>-0.39</td>
<td>-0.42</td>
<td>-0.39</td>
<td>-0.36</td>
<td>-0.39</td>
<td>-0.33</td>
</tr>
<tr>
<td>Mean survival</td>
<td>0.51</td>
<td>0.44</td>
<td>0.52</td>
<td>0.21</td>
<td>0.44</td>
<td>0.51</td>
<td>0.51</td>
<td>0.48</td>
<td>0.59</td>
<td>0.59</td>
<td>0.49</td>
</tr>
<tr>
<td>Mature size</td>
<td>-0.52</td>
<td>-0.46</td>
<td>-0.51</td>
<td>-0.27</td>
<td>-0.51</td>
<td>-0.51</td>
<td>-0.57</td>
<td>-0.50</td>
<td>-0.52</td>
<td>-0.25</td>
<td>-0.50</td>
</tr>
</tbody>
</table>

P/E = Farm profit per unit of feed ($\text{GJ}^{-1}$)

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</tr>
</thead>
<tbody>
<tr>
<td>Income from milk as a proportion of total income (%)</td>
<td>88.6</td>
<td>87.7</td>
<td>91.3</td>
<td>83.4</td>
<td>86.6</td>
<td>88.6</td>
<td>90.3</td>
<td>86.6</td>
<td>90.8</td>
</tr>
</tbody>
</table>

R = Implicit value of a replacement heifer ($)

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</thead>
<tbody>
<tr>
<td>R - value cow ($\text{GJ}^{-1}$)</td>
<td>218</td>
<td>166</td>
<td>446</td>
<td>-9</td>
<td>135</td>
<td>216</td>
<td>294</td>
<td>169</td>
<td>104</td>
</tr>
</tbody>
</table>

Table 5
Factors of energy consumed by the herd in the default scenario

<table>
<thead>
<tr>
<th>Activity</th>
<th>Maintenance</th>
<th>Growth</th>
<th>Pregnancy</th>
<th>Lactation</th>
</tr>
</thead>
<tbody>
<tr>
<td>Proportion of total energy consumed by the herd (%)</td>
<td>52.6</td>
<td>7.2</td>
<td>3.6</td>
<td>36.6</td>
</tr>
</tbody>
</table>
proportion is nearly 53% for the default scenario (see Table 5).

Survival, or herd life, is as important as body weight, and, relatively, more important than milk and fat yield. An increase of 1% survival in all age classes increases profit per cow per year by $4.05 for the default set.

Results for scenarios 1 to 8 indicate that the relative economic values are robust to moderate changes in the parameters. Scenario 3 corresponds to a high cost of volume being charged by the dairy industry, and shows significant changes in the relative importance of the traits. Essentially the income from culled stock becomes more important; the contribution of milk production to total income fell from 88.6% for the default scenario to 83.4% (see Table 4), so that survival decreases in value and mature body weight becomes less negative. Extending this to the extreme: if the negative price for milk yield was so large that income from milk production was nearly zero, the only farm profit would come from selling culled stock, and the economic values for body weight and survival would change sign.

Scenarios 4 to 7 resulted in small changes to the relative economic weights. In scenario 4 cull prices are higher. This decreases the effective cost of replacing a culled cow with a heifer, and thus reduces the economic weight for survival. This example also supports the general finding that as proportion of income from milk decreases, the economic weight of survival decreases (Table 4).

Increasing the production level (scenario 6) had little effect on relative economic weights. Even a dramatic increase (scenario 11) gave very similar relative economic weights.

From Eq. (3), economic weights are \( \frac{\partial P}{\partial x} - \frac{\partial P}{\partial x(P/F), \partial P/\partial A \text{ and } \partial P/\partial A} \) are constants, but in scenario 7 where cows are bigger but produce no more milk, \( P/F \) is reduced. Consequently, the economic weight of \( A \) is less negative. However, the standard deviation of \( A \) is assumed proportional to the mean, so that the standardised economic weight of \( A \) is more negative in scenario 7 than in the default scenario.

In the 'Jersey' scenario (8) the term \( P/F \) is greater than in the default, hence the economic weight of mature size is more negative until it is standardised by the small genetic standard deviation, when it becomes slightly less negative than in the default scenario. Because a greater proportion of income comes from milk production, the standardised economic weight of survival is increased.

Early survival, say from lactations 1 to 4, clearly has higher economic weight than survival in later age classes (see Table 6). The contribution of the economic weights of survival in the first 4 age classes to the economic weight of the sum of all survival traits is 75%. Increasing the survival in later lactations while keeping survival in early lactations constant has little effect on profit because of the small change in overall replacement rate. Using Eq. (3) the economic weight for survival traits can be split into 2 parts – i.e. change in profit (\( \delta P/\delta S \)) and change in feed 'costs' (\( \delta F/\delta S(P/F) \)). Eq. (8) for \( \delta P/\delta S \) is composed of 4 contributions to income – milk, cull cows, culled calves and costs. The total economic weight for each survival trait has been divided into these 4 components by subtracting changes in feed costs for production from milk income, change in feed costs for maintenance and growth of milk cows from cull cow income, and changes in feed costs for rearing replacements from costs. Increasing survival rates causes milk income per milking cow to increase because there are more mature cows and less heifers in the herd (ie. AC increases). For a fixed number of milking cows the number of replacements needed is decreased which reduces costs, the number of calves for sale increases but the number of cull cows for sale decreases. These effects are quantified in Table 6. In relative terms, 40% of the total economic weight for survival comes from increased milk income per milking cow but this percentage is lower for early survival traits and higher for later survival.

4. Discussion

4.1. Calculation of economic weights

Our derivations are based on the constraint that total feed supply is constant. For a pasture based production system this is close to the recommendation of Smith et al. (1986) that the total costs should be held constant, since total costs are almost proportional to feed requirements. This constraint is less appropriate for non-pasture based production systems, because then non-feed costs, such as housing costs, may make up a significant proportion of total costs. However, the results may still
be relevant to non-pasture based production, because, provided that mean profit is close to zero, the constraint used does not affect the economic weights (Brasncamp et al., 1985). Of course in this case the real costs per cows would be higher than for pasture based production, as used in scenario 10.

4.2. Relative economic weights of production traits

Relative economic values for milk, fat, and protein yield for the default set in Table 4 are similar to previous studies (Dommerholt and Wilmink, 1986; Beard, 1988; Gibson, 1989b; Harris and Freeman, 1993), with milk and fat equally important (but economic weight opposite of sign) and protein roughly 2.5 times as important as either milk or fat.

4.3. Relative economic weight of mature body size

The relative importance of (mature) body weight has been pointed out before (Morris and Wilton, 1977; Goddard, 1985, VanRaden, 1988; Ahlborn and Dempfle, 1992), but, other than in New Zealand, this trait is usually ignored in practical breeding programmes for dairy cattle. Inclusion of body weight in selection objectives and criteria will not necessarily cause mature weight to decrease but, given a positive genetic correlation between weight and production, it might prevent body weight from increasing. Groen (1989a and b) calculated economic weights for milk production traits and mature body size. From his results a standardised economic weight of body size relative to that of protein yield was found to be approximately 0.3 in the case of no output limitations (Groen, 1989a), and 0.7 in the case the total supply of roughage was fixed (Groen, 1989b). The latter constraint is very similar to our constraint of a fixed total feed supply.

4.4. Economic weight of survival relative to that of milk production

The relative importance of milk production and a measure of longevity has been reported before. For some recent analyses, the standardised ratio of economic weights for longevity/milk production was found to be 0.4 to 0.6 (Burnside et al., 1984), 0.4 (Goddard, 1987), 0.3 (Allaire and Gibson, 1992), and 0.3 (Beard, 1992). Harris and Freeman (1993) calculated a ratio of economic weights (not standardised) for herd life (days) and protein yield of approximately 0.06. Standardising the results of Harris and Freeman (1993), using a genetic coefficient of variation of 9% for protein yield and a genetic standard deviation of 0.50 for herd life (years), resulted in a standardised ratio of economic weights for (herd life)/(protein yield) of 0.2 for their average parameters. However, differences among these values and the values from Table 4 arise from variations in assumptions regarding mean (production) levels, assumptions regarding costs and returns, and from differences in the definition of traits.

Most studies (Burnside et al., 1984; Goddard 1987; Allaire and Gibson, 1992; Beard, 1992) use ‘milk production including the contribution of fat and protein’ (MI) for comparison with a measure of longevity. In our default scenario, the economic weight (EW) of MI (with 4% fat and 3.2% protein) is, $-2.042 + 0.04 \times 1.095 + 0.032 \times 3.514 = \$0.11\ per\ liter.$
The standardised economic weight (\(\text{EW}_{\text{stand}}\)), using the genetic standard deviation calculated from Table 3, is approximately $0.11 \times 5000 \times 0.19 \times 0.5 = 52.25$. This compares to a standardised economic weight of protein of $3.514 \times 160 \times 0.18 \times 0.5 = 50.60$. Hence, for the production traits, standardisation to either MI or protein yield gives similar results.

Different measurements of longevity were used in the studies cited previously. Burnside et al. (1984) used 'stayability' (in %) with a heritability of approximately 0.035. Goddard (1987), Allaire and Gibson (1992) and Beard (1992) used herd life (in years), all with heritabilities of 0.05. Harris and Freeman used herd life in days. In the present study survival was used, with a heritability of 0.025 (Table 3). In Appendix 4 the relationship between genetic parameters for herd life and survival are derived, and it is shown that there can be three to five-fold differences in the heritability for herd life and survival if survival in different age classes is (genetically) the same trait.

Allaire and Gibson (1992) point out that 'contrary to common understanding', the economic weight for herd life (adjusted for milk production) is essentially independent of feed cost when the cost of replacing a culled cow with a heifer is held constant. Harris and Freeman (1993) conclude that increasing feed cost causes the economic weight of herd life to increase because the cost of rearing a replacement heifer is increased. Our assumption of holding total feed and, therefore, total feed cost constant, directly results in economic weights being independent of feed cost.

Given the assumptions of Allaire and Gibson (1992), and ignoring the effect of herd age distribution on mean milk yield, they show that the ratio of economic weights for milk production and herd life is [the annual total fixed costs per cow] / [kg milk per cow per year] divided by [the average depreciation cost per cow per year] / [year of herd life]. However, their numerical examples are based on the assumption that genetic standard deviations for milk yield and herd life remain constant when the means change. In that case the ratio of standardised economic weights for milk yield and herd life depends on the their means. Our assumptions are much closer to implying that the genetic coefficient of variation remains constant. In that case Allaire and Gibson's (1992) ratio of standardised economic weights is simply proportional to [total fixed cost per year] divided by [depreciation cost per year of herd life]. Depreciation cost corresponds to our definition of the implicit value of a replacement heifer minus the salvage value of a culled cow (see Table 4). Qualitatively the results in Table 4 follow the pattern predicted by Allaire and Gibson (1992), but the economic weights are not very sensitive to changes in annual costs per cow.

4.5. Involuntary vs. voluntary culling

The (herd) model presented in this study is a simple one. For example, we have assumed implicitly that all culling is involuntary (\(V_i = 1\)), which is not the case. Beard (1992) used a much more detailed herd model to calculate relative economic weight for milk production (MI) and the involuntary part of longevity (LI). The standardised ratio of economic weights for LI/MI was approximately 0.3 (Beard, 1992). Converting our parameterisation to that of Beard (1992), using the derivation from Appendix 4, and assuming his genetic standard deviation of 0.67 for LI, gives a ratio of LI/MI of 0.3 for the default scenario. A simple approximation also shows that, for high levels of survival and a relative high proportion of involuntary culling, as, for example, in Australia, the standardised economic weights for survival and the involuntary part of survival are similar:

As before, \(S_i = S^* V_i\). Assume that \(S_{\text{milk}} = S^*\), \(V_i = V\), and that \(h^2(S) = h^2(S^*)\). Hence, \(S = S^* V\). Then, after some algebra, \(\text{EW}_{\text{stand}}(S^*) = \text{EW}_{\text{stand}}(S) \left(1 - \frac{V - S}{1 - S}\right)^2\). The term \(1 - \frac{V - S}{1 - S}\) is the proportion of culling which is due to involuntary culling. For \(S = 0.8\) and \(V = 0.94\), this proportion is 0.7, and \(\text{EW}_{\text{stand}}(S^*) = 0.84 \text{ EW}_{\text{stand}}(S)\). These approximations imply that the standardised weights for survival in Table 4 should be reduced by approximately 10–20% if 30% of culling is voluntary.

5. Conclusions

We conclude that our derivation for economic values for the most important components of profit is relatively easy to implement, and that the method is robust to changes in input parameters. The present method will be used to calculate customised selection indices to select bulls for farm profitability.
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Appendix 1: Growth equation

Following, among others, Van Arendonk (1985) and Beard (1992), we assume that the weight at time t, W(t), is

\[ W(t) = A h(t), \text{ with } h(t) = [1 - (1 - (p_A)^{1/3}) \exp(-k_A t)]^3 \]

with A and \( p_A \) mature body weight and birth weight as proportion of mature body weight respectively. Time t is in days from birth, and \( k_A \) is a constant. It follows that,

\[ \frac{\partial W(t)}{\partial A} = h(t) \]

Appendix 2: ME for maintenance and growth

Using formulas from SCA (1990), we can write the energy requirements for maintenance and growth per age group as,

\[ ME_m(i) = f_1(W_i) = k_s + k_a W_i \]
\[ ME_g(i) = f_2(LWG_i, W_i) = k_7 \cdot \text{LWG}_i \left( k_s + k_a \frac{1 + \exp(-6(W_i/A - 0.4))}{1 + h_s(i)} \right) \]

where LWG_i is the liveweight gain for age group i, and \( k_s \) and \( k_a \) constants. Note that \( h_s(i) \) is not a function of \( A \) but only of time (t or age group i), because \( W_i/A \) is a function of h(t).

Hence,

\[ \frac{\partial f_1(W_i)}{\partial A} = f_2(W_i) = k_s \frac{\partial W_i}{\partial A}, \text{ and} \]
\[ f_2(LWG_i, W_i) = (\partial \text{LWG}_i/\partial A) k_7 \left( k_s + k_a \frac{1 + h_s(i)}{1 + h_s(i)} \right) \]
\[ = \frac{\partial W(t_{i+1})}{\partial A} - \frac{\partial W(t_i)}{\partial A} \]

Appendix 3: The implicit value of a replacement heifer

The value of a replacement heifer can be calculated as the profit that could have been obtained if the same amount of feed had been used by the milking herd. Conceptually assume that the herd is split into two enterprises: one milking enterprise (with subscript m), and one rearing enterprise (with subscript r). The milking enterprise sells surplus calves to the rearing enterprise, and buys back replacement heifers. Define,

\[ \text{Enterprise:} \begin{array}{l} \text{Milking} \\ \text{Rearing} \end{array} \]

\[ \begin{array}{l} \text{Income} \\ \text{Cost} \\ \text{Feed consumed} \end{array} \begin{array}{l} I_m \\ C_m + RN_r \\ F_m \end{array} \begin{array}{l} RN_r \\ C_r \\ F_r \end{array} \]

where \( N_r \) is the number of replacement heifers and \( R \) is the value. \( I_m \) includes the value of selling milk, surplus calves and cull cows, and \( C_r \) includes the cost of buying the surplus calves. The profit per feed consumed from both enterprises is equal if \( [I_m - C_m - RN_r] / F_m = [RN_r - C_r] / F_r \). Hence the implicit value of a replacement heifer (\( R \)) is,

\[ R = \frac{[p_m C_r + p_r (I_m - C_m)]}{N_r}, \text{ with } p_m \]
\[ = F_m / (F_m + F_r), p_r = F_r / (F_m + F_r). \]

Appendix 4: Relationship between genetic parameters for herd life and survival

Herd life (\( L \)) is a function of survival scores \( S_i \), where \( S_i \) is the proportion of animals surviving from age i to i + 1. Because here we are only interested in survival in the milking herd, we ignore culling up to age group 2. Hence, per definition, \( S_1 = 1 \). The distribution of \( L \) is,
\[ E(L) = 1(1-S_2) + 2S_2(1-S_3) + 3S_2S_3(1-S_4) + ... \\
= 1 + S_2 + S_2S_3 + S_2S_3S_4 + ... \\
= \sum_{j=1}^{\infty} \pi \prod_{k=j}^{\infty} S_k \\
E(L^2) = 1(1-S_2) + 4S_2(1-S_3) + 9S_2S_3(1-S_4) + ... \\
= 1 + 3S_2 + 5S_2S_3 + 7S_2S_3S_4 + ... \\
= \sum_{j=1}^{\infty} (2j-1) \pi \prod_{k=j}^{\infty} S_k \\
\]

And \( \nu(L) = E(L^2) - [E(L)]^2 \)

As before, let \( S_i = S^*_i \) \( V_i \) with \( S^*_i \) survival for involuntary culling, and \( V_i \) the survival for voluntary culling.

\[ \frac{\partial L}{\partial S^*_i} = \left( \frac{\partial L}{\partial S_i} \right) \left( \frac{\partial S_i}{\partial S^*_i} \right) = \left( \frac{\partial L}{\partial S_i} \right) V_i \]

If we assume that all \( S^*_i \) are the same trait \( \left( v(S_i) = v_i(S_j) = \text{cov}_g(S_i,S_j) \right) \), then the effect of a genetic change in \( S^*_i \) is,

\[ \frac{\partial L}{\partial S^*_i} = \sum \frac{\partial L}{\partial S_i} \frac{1}{S^*_i} - S_2/S^*_i + S_2S_3(1/S^*_i) + ... \\
+ 1/S^*_i + S_2S_3(1/S^*_i + 1/S^*_i + 1/S^*_i + ... + 1/S^*_i) + ... \\
\]

and the genetic variance of herd life due to variation in involuntary culling is,

\[ \nu(L) = \left( \frac{\partial L}{\partial S^*_i} \right)^2 v_i(S^*_i) \]

For the special case where all \( S_i = S \) and all \( S^*_i = S^* \), previous equations simplify to,

\[ \nu(L) = E(L^2) - [E(L)]^2 = (1 + S)/(1 - S)^2 - 1/(1 - S)^2 = L/(L - 1), \]

\[ \frac{\partial L}{\partial S^*} = V \left( \frac{L}{S^*} \right), \text{ and} \]

\[ h^2(L) = h^2(S^*) \frac{(1 - S^*)/S^*}{(L - 1)/L} \]

So, if, for example, \( V = 1 \) and \( S = 0.8 \), then \( h^2(L) = h^2(S^*) \frac{(1 - 0.8)/0.8}{(0.2)/0.8} = 5 \), \( h^2(S^*) \). If \( V = 0.94 \) and \( S = 0.80 \) (i.e. 70% of culling is involuntary), \( h^2(L) = 3.5 \) \( h^2(S^*) \). In both cases the mean herd life is 5 years and its phenotypic standard deviation 4.5 year. Although equal survival in all age classes is only an approximation of the observed survivals, these results suggest that the heritability of herd life is 3 to 5 times that of survival.

References


Résumé


Le but de l'étude a été de calculer les poids économiques pour les caractères de production laitière (production de lait, de matière grasse et de matière protéique), longévité, et taille adulte pour des systèmes basés sur le pâturage. Les poids économiques ont été trouvés en modélisant un troupeau en en différenciant la fonction de profit par rapport aux caractères d'intérêt. Le redimensionnement a été ensuite pris en considération en supposant que l'offre alimentaire totale au niveau de la ferme était constante et à un niveau économiquement optimal. Les poids économiques relatifs ont été exprimés par unités d'âˆšœtype génétique. Pour tester la robustesse de la méthode de calcul des poids économiques, ceux-ci ont été calculés pour 11 jeux différents de paramètres en faisant varier les niveaux de production, la longévité, les coûts et les recettes. La quantité de matière protéique a eu le poids économique le plus élevé, suivie par la longévité et la taille adulte. Les poids économiques de ces deux caractères ont été approximativement de 0.5 et de −0.5 respectivement, en donnant le poids 1 à la quantité de matière protéique. La quantité de matière grasse et la quantité de lait ont eu un poids approximatif de 0.4 et de −0.4 respectivement. Les poids économiques ont été robustes aux changements de paramètres, et on a conclu que la méthode de calcul des poids économiques, en utilisant un modèle troupeau et une contrainte sur l'apport énergétique total, a été adapté aux systèmes de production basés sur le pâturage.

Kurzfassung
