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A Note on Computing the Chi-Square Noncentrality Parameter for Power Analyses

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Power calculations for a variety of research designs used in behavior genetics require the determination of a chi-square noncentrality parameter. For many purposes published tables are adequate, but for cases where they are not we provide a graph for approximating the required parameters for up to 300 degrees of freedom and illustrate a straightforward procedure for accurate determination using a widely available SAS function.

KEY WORDS: power calculations; noncentral chi-square.

There is increasing use of the noncentral chi-square distribution for assessing the power of particular survey designs to reject inappropriate genetic and environmental models. Examples include studies of the power of the classical twin design to detect additive and nonadditive genetic variation (Martin *et al.*, 1978), of the twin-family design to discriminate different models of mate selection (Heath and Eaves, 1985), and of biological and cultural inheritance (Heath *et al.*, 1986), of a familial vs. sporadic classification to detect etiological heterogenity (Eaves *et al.*, 1986), and of longitudinal twin studies to resolve alternate models of development (Hewitt *et al.*, 1988).

The procedure followed is to generate "observed" statistics under some "true" model and then to obtain the expected statistics under a false model which we would like to know our power to reject; we are

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especially interested in the sample sizes required to give adequate power. Assuming that our data will be of an appropriate kind, e.g., multivariate normal if we are modeling variance–covariance matrices, we can obtain the chi-square goodness-of-fit statistic for $s - p$ degrees of freedom, where s is the number of observed statistics and p is the number of parameters in our false model. Since in large samples of this statistic is directly proportional to the sample size (Jöreskog, 1969), we first determine $\chi^2_{s-p} = \lambda'$ for an arbitrary sample size, say 100. Providing now that the degrees of freedom, $s - p$, are not more than 100, we may use tables based on the noncentral chi-square distribution (Pearson and Hartley, 1972, Table 25) to determine the value of λ to ensure rejection of our false model at the 100α% level of significance in a given proportion, β, of studies. The required sample size is then $N \approx \lambda/\lambda' \times 100$.

This procedure may be used for overall tests of a false model or for likelihood-ratio comparisons between alternative models where these can be arranged hierarchically. In such cases the published tables of λ are usually quite adequate.

However, in the context of structural equation modeling we may well encounter situations where our data matrices are large, $s - p$ is greater than 100, and there is no obvious alternative model with which a given one should be compared hierarchically. An example is in the resolution of transmission and common factor hypotheses for developmental continuity (Hewitt *et al.*, 1988; Boomsma and Molenaar, 1987). In such cases the published tables cannot be consulted directly to obtain the required λ. This is equally the case for likelihood-ratio comparisons of factor models where the number of variables, and hence the number of factor loadings per factor, is greater than 100. The following approach is now suggested.

The power function is defined as

$$\beta(\alpha,\mathrm{df},\lambda) = \int_{\chi^2_{1-\alpha}}^{\infty} f\,(\chi'^2 \mid \mathrm{df},\lambda)d\chi'^2,$$

where χ'^2 is the noncentral chi-square p.d.f. (Pearson and Hartley, 1972). Evaluation of this function is implemented in the SAS statistical package, Version 5 (SAS Institute Inc., 1985, p. 261) using the PROBCHI function:

$$\beta = 1 - \mathrm{PROBCHI}(\chi^2_{1-\alpha},\mathrm{df},\lambda).$$

To obtain $\chi^2_{1-\alpha}$ we may use published tables or the approximation

$$\chi^2_{1-\alpha} = \tfrac{1}{2}[\sqrt{2(\mathrm{df}) - 1} + z_{1-\alpha}]^2,$$

where z is the normal deviate.

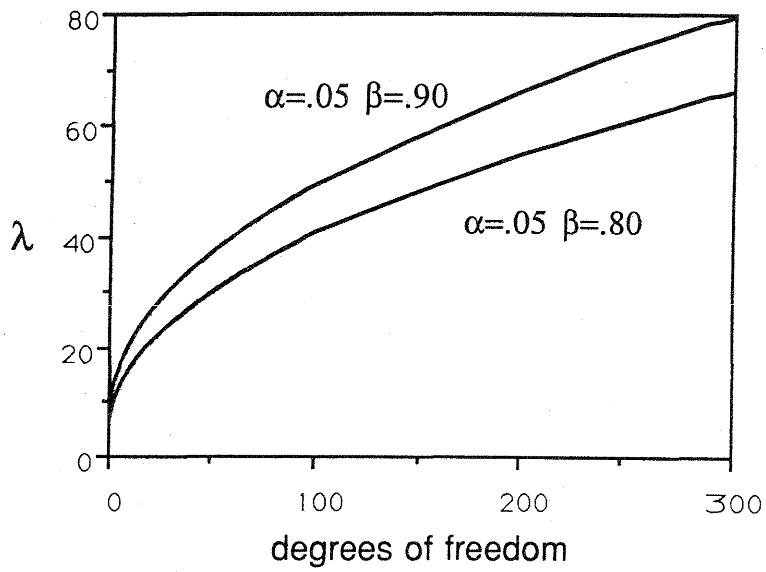

Fig. 1. Noncentrality parameter λ as a function of degrees of freedom.

Figure 1, which is based on the above procedure, may be used to obtain a graphical approximation to the required λ, and then a short numerical search undertaken to find a value of λ which gives the required β to satisfactory accuracy. An example is given in the Appendix. Analogous procedures can be used with the noncentral *F* and *t* distributions available in the SAS statistical package.

APPENDIX: AN EXAMPLE

It is required to find the noncentrality parameter λ to ensure rejection at the 5% level of significance in 90% of studies of a false model with 152 degrees of freedom.

Given $\alpha = 0.05$, $\beta = 0.90$, df $= 152$.

Now

$$\chi^2_{1-\alpha} = \tfrac{1}{2}[\sqrt{2(\mathrm{df}) - 1} + z_{1-\alpha}]^2,$$

$$\chi^2_{0.95} = \tfrac{1}{2}[\sqrt{2(152) - 1} + 1.645]^2$$

$$= 181.49.$$

From Fig. 1, $\lambda \approx 60$. Using SAS for function PROBCHI,

$$\beta = 1 - \mathrm{PROBCHI}(181.49, 152, 60)$$

$$= 0.9095$$

We now try various values of λ until we obtain a satisfactory approximation to $\beta = 0.90$.

In our case we proceeded as follows.

Step	λ	β
1	60	0.9095
2	59	0.9024
3	58	0.8950
4	58.9	0.9017
5	58.8	0.9009
6	58.7	0.9002
7	58.69	0.90017
8	58.68	0.900098
9	58.67	0.900024
10	58.66	0.899950
11	58.65	0.899876

We took $\lambda = 58.67$ as satisfactory.

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